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2007 J. Phys.: Condens. Matter 19 125217

(http://iopscience.iop.org/0953-8984/19/12/125217)

IOP Publishing — Journal of Physics: Condensed Matter

J. Phys.: Condens. Matter **19** (2007) 125217 (14pp) doi:10.1088/0953-8984/19/12/125217

# Ionizing radiation from superconductors in the theory of hole superconductivity

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Received 26 July 2006, in final form 1 September 2006
Published 6 March 2007
Online at stacks.iop.org/JPhysCM/19/125217

**Abstract**
We point out that large superconducting bodies described by the theory of hole superconductivity will emit ionizing radiation in non-equilibrium situations. This remarkable prediction, involving an energy scale a factor of $10^{12}$ larger than the low energy scale usually associated with superconductivity, is unique to the theory of hole superconductivity. The phenomenon is a consequence of the macroscopic inhomogeneous charge distribution predicted to exist in superconducting bodies, and the resulting intrinsic macroscopic spin currents in the superconducting state in the absence of applied fields. For superconducting bodies of sufficiently large size, the speed of the spin–current carriers approaches the speed of light, and in addition real electron–positron pair production is expected to occur in the interior. When the superconducting state is destroyed, electromagnetic radiation with frequencies up to 0.511 MeV/$\hbar$ should arise from bremsstrahlung and electron–positron annihilation. In support of this rather unconventional theory we point out that it is the only existing theory that proposes explanations for two fundamental universal effects associated with superconductivity: the Meissner effect and the Tao effect.

## 1. Introduction

The theory of hole superconductivity proposes that charge asymmetry is at the root of the phenomenon of superconductivity [1], and that this charge asymmetry manifests itself at a macroscopic level in the fact that superconductors expel negative charge from their interior towards the surface [2]. Because superconductivity is a macroscopic quantum phenomenon, it is natural to expect that the macroscopic quantum state that minimizes the *total* energy will *not* have the homogeneous charge distribution associated with minimum *Coulomb* energy of the electrons. No other theory of superconductivity has so far considered this possibility, possibly for the following reason: any charge inhomogeneity necessarily distinguishes positive and negative charge, and all other theories of superconductivity are electron–hole symmetric and hence blind to the sign of the electric charge. However, experiments that have detected the 'London moment' [3] and the gyromagnetic effect [4] clearly show [5] that superconductors know about the signs of electric charge.

It may be objected that electrostatic fields inside superconductors are energetically costly and for that reason cannot exist. However, this argument is faulty. It should be remembered that excluding magnetic fields from the interior is also energetically costly, yet superconductors manage to do it. The cost is paid by the superconducting condensation energy, and we argue that it is the same for the electrostatic energy cost. For example, as shown in [2], a volume energy of the order of the superconducting condensation energy for niobium will compensate for the energy cost of electric fields of the order of $10^6$ V cm$^{-1}$.

At a microscopic level, the transition to superconductivity in the theory of hole superconductivity is associated with a lowering of the electron effective mass, or 'undressing' [6], which has been described by a variety of electronic model Hamiltonians with and without auxiliary boson degrees of freedom [7, 8]. The 'undressing' phenomenology is supported by a variety of experimental observations in high $T_c$ superconductors [9–11]. The theory of hole superconductivity is proposed to describe all superconductors [12]; however, the undressing phenomenology is expected to be much less apparent in low $T_c$ superconductors.

No definitive experimental evidence for or against the theory of hole superconductivity exists so far; however, several experimental checks have been proposed that appear experimentally straightforward and hopefully will be performed soon [1, 13]. Furthermore, we have recently shown that the remarkable 'Tao effect' [14], where superconducting grains aggregate into spherical shapes upon application of electric fields, has a natural explanation within this theory [15]. No other explanation for the Tao effect has been proposed so far within the conventional [16] nor any other unconventional theory of superconductivity. In addition, the Meissner effect has a natural explanation within this theory [17], and not within any other proposed theory of superconductivity.

Qualitatively, the theory of hole superconductivity predicts that superconducting bodies look like 'giant atoms' [18], with excess negative charge near the surface and excess positive charge in the interior. There is approximately one extra electron every $10^6$ atoms near the surface [2]. The fact that the size of these giant atoms is at the experimentalist's disposal allows for a remarkably simple check of the theory. Namely, it has been predicted in the context of atomic physics that for superheavy atoms or molecules spontaneous electron–positron pair production will occur [19], when the binding energy of a K-shell electron becomes equal to twice its rest mass. Analogously, we are led to the startling conclusion that *electron–positron pair production should also occur in the interior of superconductors of sufficiently large size* [20]. And unlike real atoms, the possibility of achieving this regime appears to be well within reach. Annihilation of electron–positron pairs under suitable non-equilibrium conditions should give rise to 0.511 MeV $\gamma$-ray emission.

The theory also predicts that macroscopic spin currents exist in the ground state of superconductors [21], with some electrons moving at speeds approaching the speed of light in macroscopic samples [22]. These electrons should give rise to high frequency bremsstrahlung when the superconducting state is destroyed and the spin current stops, with photon energies up to 0.511 MeV for large samples. Ionizing radiation is unexpected within any other theory of superconductivity, and should be experimentally detectable.

We call attention to the fact that this ionizing radiation could constitute a dangerous health hazard to humans in the vicinity. Thus the determination of whether the theory is correct or not acquires an urgency that goes beyond the academic interest of deciding between competing theoretical viewpoints, since with increasing use of superconductors in society this unexpected effect could have potentially harmful consequences.

## 2. Pair production in superheavy atoms

Shortly after the introduction of Dirac theory it was pointed out by Sauter [23] that in a sufficiently strong electric field electron–positron pair creation from the vacuum should take place. For an atom, an electron in the field of a point nucleus of charge $Z|e|$ has binding energy

$$E_b = -13.6eVZ^2 \tag{1}$$

neglecting relativistic effects. It is generally believed that when the binding energy becomes degenerate with the Dirac continuum of negative energy states, i.e. $E_b = -m_e c^2$, spontaneous pair production will occur [19] ($m_e$ = electron mass). From equation (1), the condition for this to occur is $Z > 274$. In fact, when relativity is taken into account the 1s state for a point nucleus becomes unstable already for $Z > Z_c = 137$ in Dirac theory. This can also be simply seen from a Bohr atom model [24], using the relativistic equations

$$pv = \frac{Ze^2}{r} \tag{2a}$$

$$p = \gamma m_e v \tag{2b}$$

($\gamma = 1/\sqrt{1 - v^2/c^2}$) which together with angular momentum quantization $pr = \hbar$ lead to the condition

$$\frac{pc}{\sqrt{p^2c^2 + m_e^2c^4}} = \frac{Ze^2}{\hbar c} \tag{3}$$

so that for $Z = Z_c = \hbar c/e^2 = 137$, $p \to \infty$ and $r \to 0$. Solution of the Dirac equation for a nucleus of finite size shows that at a critical $Z_c = 172$ spontaneous autoionization will occur for an empty shell [19]: an electron–positron pair pops out of the Dirac sea, the electron will occupy the lowest orbit and the positron is emitted. Experimentally it has been attempted to reach the supercritical region $Z > Z_c$ by colliding two heavy nuclei; however, results to date have not been conclusive [19].

## 3. Critical radius of superconductors

In a similar vein we argue that in superconductors spontaneous pair production will become possible for superconductors larger than a critical size, within the theory of hole superconductivity. As discussed in [2], the theory predicts that electrons carrying a total negative charge

$$q = 4\pi R^2 \lambda_L \rho_- \tag{4a}$$

get expelled from the interior of the superconductor towards the surface, giving rise to a negative charge density $\rho_-$ within a London penetration depth $\lambda_L$ of the surface (for simplicity we assume a sample of spherical shape of radius $R$), as shown schematically in figure 1. The charge density $\rho_-$ is given by [2]

$$\rho_- = en_s \left(\frac{10}{3}\frac{\epsilon}{m_e c^2}\right)^{1/2} \tag{4b}$$

with $n_s$ the superfluid density, $e$ the *negative* electron charge and $\epsilon$ the condensation energy per electron. It is important to note that the charge *density* $\rho_-$ is independent of the size of the sample [2]. As the size of the sample becomes larger the amount of expelled charge increases,

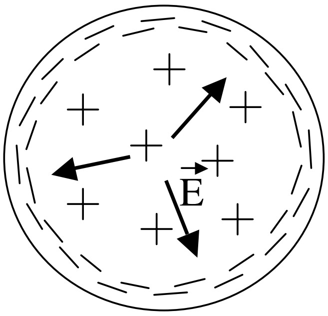


**Figure 1.** Schematic picture of charge distribution in a spherical superconductor of radius smaller than the critical radius. Excess negative charge exists within a London penetration depth of the surface, and excess positive charge in the interior.

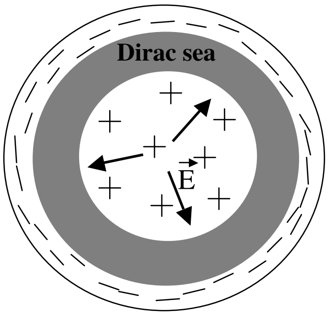


**Figure 2.** Schematic picture of charge distribution in a spherical superconductor of radius larger than the critical radius. An intermediate region between the inner positively charged region and outer negatively charged layer exists, where electron–positron pairs pop out of the Dirac sea.

and we argue that when the potential energy of an electron at the edge of the positive charge distribution becomes larger in magnitude than twice the electron rest energy

$$\frac{qe}{r} = 2m_e c^2 \tag{5}$$

pair production will become energetically favourable, as shown schematically in figure 2. Using equations (4) the condition equation (5) gives a critical radius

$$R_c = \lambda_L \left(\frac{6m_e c^2}{5\epsilon}\right)^{1/2}. \tag{6}$$

Alternatively, we may argue that the condition of dynamic equilibrium for an electron orbiting at radius $r$ in the field of a positive charge $(-q)$

$$\frac{m_e v^2}{r} = \frac{qe}{r^2} \tag{7}$$

yields that the speed $v$ approaches the speed of light $c$ for $qe/r = m_e c^2$, similar to equation (5) (note that the charge expelled $q$ increases quadratically with the size; equation (4*a*)). Of course the condition equation (7) ceases to be valid for relativistic speeds and is replaced by equation (2) with $q = Ze$, which yields $v/c = 0.91$ for $q$ given by equation (5).

As an example we consider niobium. It was estimated in [2] that the maximum electric field near the surface is $E_{max} \sim 0.77 \times 10^6$ V cm$^{-1}$, which already indicates that for samples of size of order centimetre the electron electrostatic energy becomes of the order of the electron rest mass. The London penetration depth for Nb is $\lambda_L = 400$ Å and the thermodynamic critical field $H_c = 1980$ G, from which we find condensation energy per unit volume $\tilde{\epsilon} = 1.56 \times 10^5$ ergs cm$^{-3}$, $\epsilon = 5.54$ $\mu$eV, and from equation (6)

$$R_c = 3.33 \times 10^5 \lambda_L \tag{8}$$

hence $R_c = 1.33$ cm.

## 4. Dynamical equilibrium in applied magnetic field

In the previous section we argued that for sufficiently large superconductors the electric field resulting from negative charge expulsion will lead to pair production. Here and in the next section we show that this expectation is consistent with the Meissner effect and the requirement of dynamical equilibrium for superfluid electrons.

We assume the validity of London's equation

$$\vec{\nabla} \times \vec{J} = -\frac{c}{4\pi\lambda_L^2}\vec{B} \tag{9}$$

for the screening charge current $\vec{J}$ in the presence of an applied magnetic field $\vec{B}$, with $\vec{J}$ given by

$$\vec{J} = \rho\vec{v}_\phi. \tag{10}$$

Here, $\rho$ is the superfluid transport charge density and $\vec{v}_\phi$ its azimuthal velocity induced by the applied magnetic field. In conventional London theory it is assumed that $\rho = en_s$, with $n_s$ the total superfluid charge density, independent of the volume of the sample; however, we show here that this is untenable in the present context.

In the presence of the electric field $\vec{E}$ resulting from charge expulsion (figure 1), the expelled electrons near the surface will carry a spin current even in the absence of an applied magnetic field [21], to satisfy dynamical equilibrium, with

$$\frac{m_e v_0^2}{r} = |e|E \tag{11}$$

for electrons at radius $r$ (neglecting relativistic corrections). Electrons of opposite spin will traverse their orbits with opposite velocities of equal magnitude $v_0$. When a magnetic field is applied, velocities of spin up and down electrons change according to

$$\vec{v}_\uparrow = \vec{v}_0 + \vec{v}_\phi \tag{12a}$$

$$\vec{v}_\downarrow = -\vec{v}_0 + \vec{v}_\phi \tag{12b}$$

and in particular for equatorial orbits

$$v_\sigma = \sigma v_0 + v_\phi \tag{12c}$$

so that the requirement of dynamical equilibrium for equatorial orbits is

$$\frac{m_e v_\sigma^2}{r} = |e|E + \frac{|e|}{c}v_\sigma B \tag{13}$$

which implies (to lowest order in $B$)

$$v_\phi = -\frac{e}{m_e c}\frac{Br}{2}. \tag{14a}$$

This azimuthal velocity $v_\phi$ induced by the magnetic field is much larger than what would result if $\rho = en_s$ in equation (10), namely

$$v_\phi \sim -\frac{e}{m_e c}B\lambda_L \tag{14b}$$

since from equation (9), because $B$ is non-zero only in a region of thickness $\lambda_L$ near the surface,

$$J \sim -\frac{c}{4\pi\lambda_L}B. \tag{15}$$

From equations (10), (14a) and (15) it follows that

$$\rho = \frac{m_e c^2}{2\pi\lambda_L e r} \tag{16}$$

hence the transport charge density decreases inversely with the radius $r$. Using for the London penetration depth

$$\frac{1}{\lambda_L^2} = \frac{4\pi n_s e^2}{m_e c^2}, \tag{17}$$

equation (16) becomes

$$\rho = 2en_s \frac{\lambda_L}{r} \tag{18}$$

which shows that the transport charge density approaches the conventional value $en_s$ for small samples, but is much smaller than the conventional value for samples much larger than the London penetration depth.

Now it is reasonable to assume that for macroscopic samples the transport charge density cannot be smaller than the expelled charge density $\rho_-$. From equations (4*a*) and (16) we find, setting $\rho = \rho_-$ and $r = R$

$$\frac{qe}{R} = 2m_e c^2 \tag{19}$$

i.e. the same condition as equation (5). We conclude that at the critical radius given by equation (6) the transport charge becomes equal to the excess expelled negative charge equation (4).

## 5. Interpretation of the Meissner effect

The conclusion reached in the previous section that the transport charge in macroscopic samples is only the excess charge $\rho_-$ rather than the full superfluid charge density $en_s$ also leads to an understanding of the Meissner effect. Indeed, consider cooling a superconductor in the presence of an applied magnetic field $\vec{B}$. The charge that is expelled from the interior towards the surface experiences a Lorentz force due to the magnetic field

$$\frac{d\vec{v}}{dt} = \frac{e}{m_e c}\vec{v} \times \vec{B} \tag{20}$$

and the azimuthal velocity builds up as electrons move radially outward due to this force. Using $\vec{v} = d\vec{r}/dt$ we find on integrating equation (20)

$$\vec{v}_\phi(t=\infty) = \frac{e}{m_e c}[\vec{r}(t=\infty) - \vec{r}(t=0)] \times \vec{B}. \tag{21}$$

The charge expulsion process results in a total excess negative charge

$$q = 4\pi R^2 \lambda_L \rho_- \tag{22}$$

residing in the layer of thickness $\lambda_L$ at the surface. This negative charge moved outwards upon cooling from above to below $T_c$, from an initial spherical volume of radius $(R - \lambda_L)$ to the spherical shell of inner radius $(R - \lambda_L)$ and outer radius $R$, as depicted in figure 3. As shown in figure 3, an electron initially near the centre of the sphere (denoted as 1) moves from radius $r(t=0) \sim 0$ to radius $r(t=\infty) \sim R - \lambda_L$, and in so doing acquires an azimuthal speed

$$v_{1\phi} \sim -\frac{e}{m_e c} BR \tag{23a}$$

according to equation (21). This is in agreement with the speed obtained from the condition of dynamical equilibrium with a pre-existent $v_0$, equation (14*a*). Instead, an electron initially

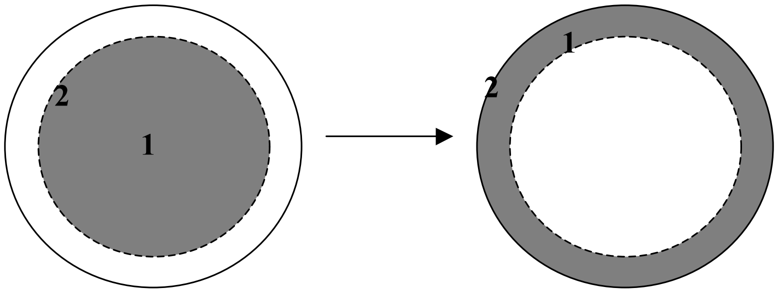


**Figure 3.** Schematic picture of the charge expulsion process. The shaded spherical negative charge distribution of radius $R-\lambda_L$ on the left becomes the outer shell of thickness $\lambda_L$ on the right. Electron at position 1, $r \sim 0$ on the left, moves to $r \sim R-\lambda_L$ on the right; electron at position 2, $r \sim R-\lambda_L$ on the left, moves to $r \sim R$ on the right.

at radius $(R - \lambda_L)$ (denoted by 2 in figure 3) ends up at the surface of radius $R$, acquiring an azimuthal speed

$$v_{2\phi} \sim -\frac{e}{m_e c} B\lambda_L \tag{23b}$$

according to equation (21), which is of the same form as the 'classical' speed equation (14*b*).

In summary, these arguments show that our conclusion in section 4 that electrons in the screening current have an excess azimuthal speed $v_\phi$ which is very large as given by equation (14*a*) is consistent with the interpretation that these electrons are expelled from the region near the centre of the sphere, and acquire this azimuthal speed as their radial position changes from $r \sim 0$ to $r \sim R - \lambda_L$ due to the Lorentz force. Instead, it is only electrons near the outer surface of the surface layer that move at the 'classical' speed, equation (14*b*), since they have only traversed a radial distance $\lambda_L$ in the process of charge expulsion. In comparison, the conventional interpretation is that the entire superfluid density contributes to the Meissner screening current with the classical speed, equation (14*b*); however, this interpretation is incompatible with the requirement [17] that the azimuthal speed arises from the Lorentz deflection of the outgoing radial motion. The conventional theory of superconductivity does not offer any explanation for the process by which the superfluid electrons acquire an azimuthal velocity when a normal metal is cooled into the superconducting state in the presence of an external magnetic field.

## 6. Interpretation of the London field of rotating superconductors

In superconductors rotating with angular velocity $\vec{\omega}$ a uniform magnetic field

$$\vec{B} = -\frac{2m_e c}{e}\vec{\omega} \tag{24}$$

exists in the interior (London field) [3, 25, 26]. This is conventionally interpreted as arising from a 'lagging' of the superfluid rotation within a London penetration depth of the surface: superfluid electrons rotate slower than the rigid rotation speed $v_\phi = \omega R \sin\theta$ by an amount [26]

$$\delta v_\phi \sim \omega\lambda_L \sin\theta \tag{25}$$

and the azimuthal current density is of order

$$J_\phi \sim e n_s \delta v_\phi \sim e n_s \omega \lambda_L. \tag{26}$$

The total current $I$ due to the superfluid electrons in a surface shell of thickness $\lambda_L$ is of order

$$I \sim 4\pi R^2 \lambda_L J_\phi \tag{27}$$

and the magnetic field due to a ring current of radius $R$ is of order

$$B \sim \frac{2I}{cR}. \tag{28}$$

Replacing equation (27) in (28) and using equation (17) for the London penetration depth, the magnetic field equation (24) results.

The problem with this argument is that it does not provide a rationale for why the superfluid electrons near the surface would suddenly 'lag behind' when a rotating normal metal is cooled into the superconducting state. Instead, in our scenario the charge that 'lags behind' is the transport charge density equation (16), which for macroscopic samples is the same as the excess charge density $\rho_-$ residing within a London penetration depth of the surface. Using

$$J_\phi = \rho \delta v_\phi \tag{29}$$

together with equations (16), (27) and (28) and demanding that the resulting magnetic field $B$ be the London field equation (24) leads to

$$\delta v_\phi \sim \frac{\omega R}{2} \sin\theta. \tag{30}$$

If the transport charge density was expelled from the interior of the superconductor at an average radius $R/2$, it will lag the azimuthal velocity at the surface by precisely the amount in equation (30). Hence our point of view provides a simple rationale for how the London field develops when a rotating normal metal is cooled into the superconducting state: the electrons near the surface that suddenly 'lag behind' are electrons that were expelled from deep inside the superconductor, where much smaller rigid rotation speeds prevail.

## 7. Samples larger than critical: the intermediate layer

Consider now a sample of radius $R$ that is larger than the critical radius $R_c$ given by equation (6). Let $q_1$ be the charge expelled from the inner region, that satisfies

$$\frac{q_1 e}{R_c} = 2m_e c^2, \tag{31}$$

and let $(q_1+q_2)$ be the total negative charge in the outer layer of thickness $\lambda_L$, that is responsible for the charge transport. According to the condition of dynamical equilibrium derived in the previous section, equation (19), we have

$$\frac{(q_1+q_2)e}{R} = 2m_e c^2 \tag{32}$$

hence from equations (31) and (32) we conclude that the electric potential is constant in the intermediate region $R_c < r < R$, consequently that *no electric field exists in the intermediate region*. Clearly, screening of the electric field has occurred through creation of *real electron–positron pairs*. The positrons reside at the outer surface of the intermediate layer, and the newly created electrons move out and add to the surface layer negative excess charge $\rho_-$, now satisfying

$$q_1 + q_2 = 4\pi R^2 \lambda_L \rho_-. \tag{33}$$

Hence the positive charge ($-q_2$) is due to *real positrons* created from the Dirac vacuum, and its magnitude is

$$|q_2| = |q_1|\left(\frac{R}{R_c} - 1\right) \tag{34}$$

with $q_1$ given by equation (4) with $R = R_c$ as given by equation (6).

For the case of Nb, $\rho_- = 0.017$ C cm$^{-3}$ [2], yielding $|q_1| = 1.5 \times 10^{-6}$ C. For example, in a sample of radius $R = 2R_C = 2.66$ cm, equation (34) predicts that $9.4 \times 10^{12}$ real electron–positron pairs created from the Dirac vacuum exist!

Finally, we note that in the intermediate region there is no net electric field, and that because of the electron–positron pair creation the system cannot be described with a wavefunction with a fixed number of particles, but rather requires a description that allows for superposition of charge-neutral states with different numbers of particles. This is precisely the physical situation described by the conventional BCS wavefunction.

## 8. Pair production from externally applied electric field

We have seen in the previous section that creation of real electron–positron pairs is expected to occur for large superconducting samples when the internal electric field exceeds a critical value, to prevent the internal field from becoming larger. Here we show that application of an external electric field is another mechanism leading to pair production.

Indeed, the relation between charge density $\rho(\vec{r})$ and electrostatic potential $\phi(\vec{r})$ in our theory is [2]

$$\rho(\vec{r}) - \rho_0 = -\frac{1}{4\pi\lambda_L^2}[\phi(\vec{r}) - \phi_0(\vec{r})] \tag{35}$$

where $\rho_0$ and $\phi_0(\vec{r})$ are the interior positive charge density and the associated electrostatic potential respectively. Under an externally applied field the induced charge density is

$$\rho_{\text{ind}}(\vec{r}) = -\frac{1}{4\pi\lambda_L^2}\delta\phi(\vec{r}) \tag{36a}$$

where $\delta\phi(\vec{r})$ is the change in the total electrostatic potential due to the applied electric field. Equation (36*a*) shows that externally applied electric fields are screened over a London penetration depth, just as magnetic fields. Using equation (17) for $\lambda_L$,

$$\rho_{\text{ind}}(\vec{r}) = -n_s e\frac{e\delta\phi(\vec{r})}{m_e c^2}. \tag{36b}$$

In contrast, the induced charge density in a normal metal when an external electric field is applied is

$$\rho_{\text{ind}}(\vec{r}) = -\frac{1}{4\pi\lambda_{TF}^2}\delta\phi(\vec{r}) \tag{37a}$$

with the Thomas–Fermi screening length given by (for free electrons)

$$\frac{1}{\lambda_{TF}^2} = \frac{6\pi n e^2}{\epsilon_F} \tag{37b}$$

with $n$ the electron density and $\epsilon_F$ the Fermi energy, so that equation (37*a*) is

$$\rho_{\text{ind}}(\vec{r}) = -\frac{3}{2}ne\frac{e\delta\phi(\vec{r})}{\epsilon_F}. \tag{37c}$$

Equation (37*c*) for the normal metal shows that the fractional change in the local charge density ($ne$) induced by the external field is the ratio of the energy gain per electron $e\delta\phi$ to the energy cost in putting an extra electron at the top of the Fermi distribution, $\epsilon_F$. Analogously, for the superfluid, equation (36*b*) shows that the fractional change in the local superfluid charge density induced by the external field is the ratio of $e\delta\phi$ to the energy cost in creating charges from the Dirac vacuum, $m_e c^2$. Because this cost is much greater than $\epsilon_F$, the London length is much larger than the Thomas–Fermi length. We conclude that, for the superfluid, screening of externally applied electric fields occurs through pair production from the Dirac sea, because the 'rigidity' associated with the coherence of the superfluid wavefunction over the macroscopic sample prevents screening through local shifts of the superfluid density.

Consequently, equation (36) directly furnishes the number of positrons created under application of an external electric field. The change in potential under an applied electric field $E_{app}$ that decays to zero over a distance $\lambda_L$ is

$$\delta\phi \sim E_{app}\lambda_L \tag{38}$$

giving rise to an induced charge density

$$\rho_{ind} \sim -\frac{1}{4\pi\lambda_L}E_{app} \tag{39}$$

generated by pair production over a layer of thickness $\lambda_L$. The total charge created for a sample of surface area $A$ exposed to the electric field is

$$q_{ind} \sim \rho_{ind}\lambda_L A = \frac{E_{app}}{4\pi}A \tag{40}$$

hence the number of positrons created is

$$N_p \sim \frac{E_{app}A}{4\pi|e|}. \tag{41}$$

For example, for $E_{app} = 1$ kV mm$^{-1}$ and $A = 1$ cm$^2$, $N_p \sim 5.5 \times 10^9$.

## 9. Observable consequences of pair production

We have seen in section 6 that in a spherical superconductor of radius $R$ larger than the critical radius $R_c$ given by equation (6) the amount of negative charge in the surface layer of thickness $\lambda_L$ exceeds the negative charge expelled from the interior by an amount

$$q_2 = q\left(\frac{R}{R_c} - 1\right) \tag{42}$$

with $q$ given by equation (4). This excess negative charge originates in pair creation in the intermediate layer and reflects the existence of real positrons in the intermediate layer. Of course the same phenomenon is expected in large samples of non-spherical shape. Similarly, we have argued in section 7 that when an external electric field is applied on a surface area $A$ real electron–positron pairs are created in a surface layer of thickness $\lambda_L$ to screen the applied field.

When electrons and positrons collide they annihilate and two $\gamma$-rays of energy $m_e c^2 =$ 0.511 MeV are emitted in opposite directions. Of course in a superconductor in equilibrium or under stationary conditions no such $\gamma$-ray emission is expected. We propose however that under suitable non-stationary conditions 0.511 MeV $\gamma$-rays will be emitted from large superconductors, and suggest that an experimental effort to detect this effect should be undertaken. Because it is an unexpected effect within the conventional theory of superconductivity, if this phenomenon is detected it will shed important new light onto the physics of superconductivity.

We suggest the following experimental tests. For a large superconducting sample at low temperatures, positrons should exist in dynamical equilibrium with electrons in the intermediate layer. Rapid destruction of superconductivity by heating, or application of ultrafast ultrastrong magnetic field pulses that drive the material normal, should cause electron–positron annihilation and emission of 0.511 MeV $\gamma$-rays that can be detected with appropriate detectors. Similarly, in the presence of an externally applied electric field positrons will exist which will annihilate and give rise to $\gamma$-ray emission if the electric field is suddenly switched off. We suggest that placing a superconducting sample between the plates of a capacitor and increasing the electric field until breakdown occurs will give rise to $\gamma$-ray emission when the capacitor discharges through the superconductor. Alternatively, $\gamma$-ray emission should also occur upon application of a strong rapidly varying ac electric field.

However, $\gamma$-radiation can be hazardous to humans. Exposure to 5 rad per year is usually regarded as the limit of safety, and $5 \times 10^9$ photons of energy 0.511 MeV cm$^{-2}$ of tissue is 1 rad. For the example discussed in section 6, a sample of Nb of radius $R = 2R_c = 2.66$ cm has $\sim 10^{13}$ electron–positron pairs; if each pair emits two photons when the sample goes normal this results in $2 \times 10^{13}$ photons and a radiation dose of 0.13 rad to a person 50 cm away. In cycling the sample from below to above $T_c$ several times, very quickly the limit of safety for this individual is met! For larger samples the danger becomes rapidly larger as seen from equations (4) and (34). Similarly, application of large electric fields to superconductors can result in significant numbers of electron–positron pairs being created, as discussed in section 8, and experiments with large time-varying electric fields could also result in hazardous amounts of $\gamma$-radiation.

## 10. Bremsstrahlung

In addition to 0.511 MeV radiation, we expect that high energy radiation with a broad spectral range will be emitted from superconductors that are rapidly driven normal through sudden changes in temperature or applied magnetic field.

Indeed, the electrons expelled from the interior that give rise to the outer negative charge density $\rho_-$ carry a spin current in the superconducting state. At radius $r$ the kinetic energy of these electrons is, from equation (7),

$$\frac{1}{2} m_e v_0^2 = \frac{qe}{2r} \tag{43}$$

where $q$ is the net charge inside $r$. In particular, the fastest speeds occur for electrons at the edge of the positive charge distribution shown in figure 1. If the superconductor is suddenly driven normal these electrons carrying the macroscopic spin current will stop and emit bremsstrahlung, of maximum frequency $\omega_m$ determined by conversion of the entire kinetic energy of the electron into a single photon:

$$\hbar\omega_m(r) = \frac{qe}{2r}. \tag{44}$$

Hence at the critical radius given by equation (31) we obtain from equation (44)

$$\hbar\omega_m(R_c) = 0.511 \text{ MeV} \tag{45}$$

which is the same as the photon energy resulting from electron–positron destruction. For radius $R$ smaller than the critical radius we have simply

$$\hbar\omega_m = \frac{R}{R_c} m_e c^2. \tag{46}$$

Consequently, we expect a broad spectrum of high energy radiation, with the upper limit frequency $\omega_m$ increasing with sample size: samples of radius smaller than $2 \times 10^{-4} R_c$ will

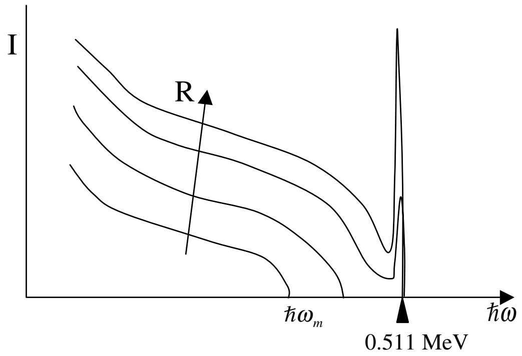


**Figure 4.** Schematic picture of the intensity of radiation versus frequency expected when a superconductor is heated from below to above $T_c$. $R$ is the radius of the superconductor for a spherical sample. For $R$ larger than the critical radius $R_c$ (equation (6)) the peak at 0.511 MeV appears.

emit in the UV ($\hbar\omega_m < 100$ eV), samples of radius up to $R = 0.2R_c$ will also emit x-rays (100 eV $< \hbar\omega_m <$ 100 keV) and samples with $R > 0.2R_c$ will in addition emit $\gamma$-rays ($\hbar\omega_m > 100$ keV), up to a maximum frequency 511 keV$/\hbar$ when the radius reaches the critical radius $R_c$. The radiation will originate predominantly from the region at distance $\lambda_L$ from the surface of the sample, where the fastest electrons in the spin current reside. When the system becomes normal, these 'undressed' electrons [27] in Cooper pairs will suddenly unbind and experience scattering by the discrete ionic potential, and emit a bremsstrahlung spectrum as given by the Bethe–Heitler formula [28]. The spectral distribution detected will depend both on the bremsstrahlung processes and on the scattering processes that occur in the path of the photon towards the surface. When the sample radius becomes larger than $R_c$, a peak will grow at the maximum frequency 511 keV$/\hbar$ reflecting the electron–positron annihilation processes. This is schematically depicted in figure 4. The radiation will be further enhanced if a *charge* supercurrent is circulating when the system is driven normal.

The integrated intensity of the bremsstrahlung radiation should be proportional to the negative charge in the outer layer shown in figure 1. Hence we expect the integrated intensity to grow proportionally to $R^2$ for samples of radius smaller than $R_c$, according to equation (4). For $R > R_c$, the negative charge density $\rho_-$ starts to decrease according to equation (16), so the integrated intensity (excluding the peak at 0.511 MeV) should grow proportionally to $R$. The peak at 0.511 MeV should increase proportionally to $(R/R_c - 1)$ according to equation (34).

We conclude from these considerations that experiments with and practical uses of large superconducting samples, as well as processes involving application of large electric fields to superconductors, are potentially dangerous. The level of ionizing radiation generated in these situations should be ascertained before they can be safely carried out in an environment where humans are in danger of exposure.

## 11. Discussion

Superconductivity has been traditionally regarded as a low energy phenomenon, because low temperatures are involved and because in the conventional theory phonons, that are low energy excitations in the solid, are thought to play the dominant role. Only recently, evidence from optical experiments in high $T_c$ superconductors [10] has suggested that higher energy scales, in the mid-infrared and visible range (of order eV), play a role at least in these materials. We have also recently suggested changes in the plasmon dispersion relation [29], which for conventional superconductors can be above 10 eV; this prediction has not yet been put to experimental test. Continuing this trend, in this paper we have suggested that energy scales relevant to superconductivity extend even much higher, to the range of millions of electron volts.

How can mega-electron-volt energies possibly be relevant for a phenomenon where the local energies involved are of the order of micro-electron volts, i.e. a factor of $10^{12}$ smaller? Qualitatively, the key lies in the quantum coherence of the superconducting state over macroscopic distances. In a sample of volume 1 $cm^3$ there are of the order of $10^{23}$ atoms in the bulk, and of the order of $10^{18}$ atoms in the surface layer of thickness $\lambda_L$. Hence a fraction $10^{-7}$ of the surface layer atoms could each display a phenomenon at an energy scale of $10^6$ eV if they are able to harness an energy of $10^{-6}$ eV from each of the atoms in the bulk. Therein lies the remarkable nature of the macroscopic quantum coherence that is the hallmark of superconductivity.

The importance of relativity in the theory of hole superconductivity was already foreshadowed early on in the lattice formulation of the theory [6], which describes pairing and superconductivity as driven by an off-diagonal Coulomb interaction term in the Hamiltonian, $\Delta t$. This term gives rise to a *reduction of the mass* of the carriers when they form a Cooper pair [31], evidence for which has recently been seen experimentally [10]. In relativity a *bound state* of two particles necessarily has a smaller mass than the sum of its constituent's masses due to the energy–mass relation $E = mc^2$.

The work discussed here also sheds new light on the meaning of the BCS wavefunction. The fact that the BCS wavefunction describes the superconducting state as a superposition of states with different number of particles has until now been regarded merely as a convenient calculational device, without physical content. Indeed, there is no *physical* reason in the conventional theory why a wavefunction describing pairing of electrons could not be described with a fixed number of pairs. Furthermore, there is something profoundly unphysical about the BCS wavefunction in the conventional context: each Cooper pair carries a mass of 1.022 MeV/$c^2$ and a charge of $2e$, so that the BCS wavefunction superposes states with widely different electric charges and energies. Why would the description of a low-energy phenomenon require the mixing of such very different states? Instead, in the present context the superposition of states with different numbers of electrons and positrons (but the same total electric charge) arises as a *consequence of the physics* and indicates that a BCS-like wavefunction that superposes different occupation number sectors is in fact *required* to describe the physical reality.

In physics, the first 'hole theory' proposed was that of Dirac [32], to deal with the negative energy states that he encountered in formulating the relativistic quantum theory of the electron. In this paper we have shown that the theory of hole superconductivity leads unavoidably to the inclusion of Dirac's holes (positrons) in the description of the superconducting state. The theory predicts that ionizing radiation with a continuum of frequencies all the way up to 0.511 MeV/$\hbar$, i.e. photons arising from electron–positron annihilation processes, will be emitted from large superconductors in non-equilibrium situations. No other theory of superconductivity predicts this startling effect, hence detection of such radiation will strongly support the theory of hole superconductivity. Of course, alternative explanations will be proposed and explored if the phenomenon is detected.

In summary: the theory of hole superconductivity explains the Tao effect, the Meissner effect, the magnetic field of rotating superconductors, the form of the BCS wavefunction, and many experimental observations in high $T_c$ cuprates that are not explained by the conventional theory. It has a solid microscopic foundation in atomic physics [33]. It makes several unique and unexpected experimental predictions that can be experimentally tested. It implies that it is important to establish as soon as possible the range of conditions where special precautions should be taken to avoid hazardous effects from superconductors to humans. It provides a unified framework to describe *all* superconductors, and suggests guidelines for the search of new superconducting compounds [34]. We suggest that all these reasons add up to a compelling case to devote an experimental effort to prove the theory right or wrong.

## Acknowledgments

The author is grateful to Joaquin Fernandez-Rossier and Frank Marsiglio for stimulating comments.

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